

Exercise 02

Energy and Entropy Competition in Micelle Formation

a) In this problem we examine some of the interesting forces that arise in the crowded environments of a cell. The water molecules that can be modelled as clathrate structures such as are shown in figure 1. In presence of a hydrophobic molecule, one of the vertices of the tetrahedron can be thought of pointing toward that hydrophobic molecule and hence all configurations facing that molecule are unavailable for hydrogen bonding. Estimate the entropy loss for each water molecule by appealing to the schematic of the tetrahedron. Use the Boltzmann equation

$$S = k_B \ln(W)$$

and the number of configurations available in the absence and presence of a hydrophobic molecule facing the water molecule.

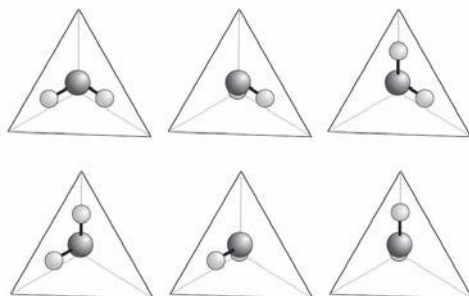


Figure 1. Orientations of water molecules in a tetrahedral network. Each image shows a different arrangement of the water molecule that permits the formation of hydrogen bonds with neighbouring water molecules. The hydrogen bonds are in the directions of the vertices in the Figure (Adapted from K. Dill and S. Bromberg, Molecular Driving Forces, New York, Garland Press, 2003)

b) Consider the case of methane and ethane, approximate them as spheres and estimate the radius of the spheres that represent the hydrophobic surface area. Estimate the number of neighbouring water molecules (approximate, using the dimensions of a face of a water tetrahedron where the length of an O-H bond is approximately 1\AA and the length of a hydrogen bond ca. 1.8\AA) and calculate the total free energy difference due to the loss in entropy. Convert your result into an interfacial energy and use units both of J/m^2 and $\text{cal/mol } \text{\AA}^2$.

Note: Van der Waals forces contribute to the stability of hydrophobic interactions by providing attractive forces between non-polar molecules in aqueous environments. Include the Van der Waals radius for Hydrogen (search it online, e.g. <https://bionumbers.hms.harvard.edu/>) in the calculation of the hydrophobic surface area.

c) Using the same approach, calculate the hydrophobic cost for an individual lipid in solution as a function of both the length and the number of tails. Consider the lipid tails as cylinders of length L and radius r and use a length of 1.4\AA per carbon.

d) Compute a model to derive the critical micelle concentration for lipid molecules. The chemical potential of a lipid in solution has two components, a term that has to do with the number of ways arranging lipid molecules in solution (first part of eq. 2, not derived here), and the interfacial energy derived in part c. This comes for the fact that the lipids are hydrophobic, limiting the entropy of the surrounding water molecules:

$$\mu_{lipid} = k_B T \ln(c \lambda_{th}^3) + 2\pi R L \gamma$$

where $\lambda_{th} = h / \sqrt{2\pi m k_B T}$ is the thermal de Broglie wavelength, c the concentration, L the length of the lipid and γ the interfacial energy associated with the loss of entropy of water molecules derived above. Assume $\mu_{micelle} = 0$. Calculate the critical micelle concentration for two different lipids in molar units.